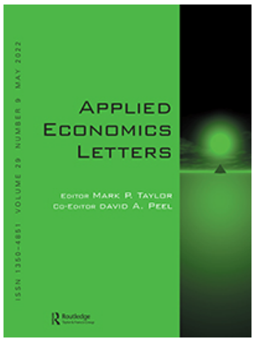

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ABSTRACT
This paper studies social interaction models with both in-group and out-group effects. The in-group effect follows the standard setup in the literature, while the out-group effect is introduced by assuming the economic outcome also depends on its out-group average value. We present a network game with limited information of outside groups that rationalizes the econometric model. We show that both effects are identified under a set of mild regularity conditions. We propose to estimate the model using the two-stage least squares (2SLS) method and establish the asymptotic normality of the estimators. The finite sample performance of the estimators is investigated through Monte Carlo simulations.

KEYWORDS
Social interaction models; in-group effects; out-group effects; 2SLS

JEL CLASSIFICATION
C31; C51

I. Introduction

Ever since the seminal work of Manski (1993), social interaction models have attracted considerable attention from both theoretical and empirical sides; see Jackson, Rogers, and Zenou (2017) and Kline and Tamer (2020) for a comprehensive review. The key feature of such models is that the economic outcome of interest is not only determined by one's own characteristics but also by his peers. For example, students' academic achievement, measured by GPA, is also affected by their friends' performance (Lin (2010)).

Motivated by the fact that many real-world networks can be further decomposed into subgroups, a large amount of literature has focused on social interaction models with group structures; see Lee (2007), Liu and Lee (2010) and Bramoullé, Djebbari, and Fortin (2009), among many others. All these studies assume that individuals can only be affected by their within-group friends. Such setting, however, can be restrictive in reality because potential group-level interaction effects are completely ignored. We illustrate this point using the same example of students' academic achievement. Suppose that all students in some city form a network. This single network can be further decomposed by treating each school as a group. It is likely that a student's GPA may not only be affected by students in his school but also by the average academic performance of students in other schools if all students need to compete together, such as taking the city-level high school entrance examination.

In this paper, we regard the social interaction effect induced by individuals outside the group as *the out-group social interaction effect*. To introduce such effect into the classic social interaction models, we assume that one's economic outcome depends not only on his friends' economic outcomes but also the average value of other groups. This setting is motivated by the observation that one may not know the situation of other groups as well as of his own group. For example, it is likely that students have more information of the academic achievement of his peers in the same school than in other schools.

We show that both *the in-group social interaction effect* and *the out-group social interaction effect* are identified under a set of assumptions that have been made in previous studies (Bramoullé, Djebbari, and Fortin 2009). To estimate the parameters of interest, we adopt the two-stage least squares estimation method developed in Kelejian and Prucha (1998) and establish the asymptotic normality of the estimators. We investigate the

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Supplemental data for this article can be accessed here.

finite sample performance of the 2SLS estimators through Monte Carlo simulations, which show they performs very well.

Our paper contributes to the literature of social interaction models by first introducing *the out-group social interaction effect*. It is noteworthy that ignoring *the out-group social interaction effect* may lead to a significant bias of *the in-group social interaction effect* because these two effects are often positively correlated in practice. We illustrate this observation based on numerical experiments in Section 4. With the model and the asymptotic results developed in this paper, one can conveniently test whether *the in-group social interaction effect* alone is enough to capture all the interaction effects in real-world network data sets, making our model an appealing choice for empirical studies.

The rest of the paper is organized as follows: Section 2 presents the econometric model and a network game as its microfoundation. Section 3 studies the identification and the 2SLS estimation of the model. Section 4 investigates the finite sample performance of the proposed estimators through Monte Carlo simulations. Section 5 concludes. The online appendix offers proofs.

Notations. For any real vector or matrix A, we use A^{T} to denote the transpose of A and A^{-1} to denote its inverse. We use A_{ij} to denote the ijth element of a matrix A. For two positive integers a and b, we let $\mathbf{0}_{a\times b}$ denote the $a\times b$ matrix consists of zeros and $\mathbf{1}_a$ denote the a-dimensional unit vector. For a sequence of random variables X_n, we let $\mathrm{plim}_{n\to\infty}X_n$ denote its probability limit, $\xrightarrow{p}$ and $\xrightarrow{d}$ denote convergence in probability and in distribution, respectively.

II. Setup

The model

Suppose we have data of a single network which consists of n individuals and K groups. We let G_k denote the kth group. In the group G_k, $k=1,\dots,K$, there are n_k individuals, so $n=n_1+\cdots+n_k$. The corresponding $n_k\times n_k$ adjacency matrices W_k are observed.[1] Without loss of generality, we let $G_1=\{1,\dots,n_1\},\dots,G_K=\{\sum_{k=1}^{K-1}n_k+1,\dots,\sum_{k=1}^{K}n_k\}$ denote the group structure and we use $G(i)$ to represent the individual i's group for $i=1,\dots,n$. Following the literature (e.g.Lee 2007; Bramoullé, Djebbari, and Fortin 2009), we assume that links only exist within groups. The social interaction model with both in-group and out-group effects is given by:

$$y_i=\lambda_1\sum_{j\in G(i)}W_{G(i),ij}y_j+\lambda_2\bar{y}_{-G(i)}+x_i^{\mathrm{T}}\beta+\in_i, \quad (1)$$

where y_i is the outcome variable of interest, x_i is a $p\times 1$ vector of nonstochastic individual-specific characteristics, $\bar{y}_{-G(i)}$ is the average value of the economic outcome outside the group $G(i)$, i.e. $\bar{y}_{-G(i)}=1/(n-n_{G(i)})\sum_{j\notin G(i)}y_j$, $W_{G(i),ij}$ is the ijth element of the adjacency matrix of the group $G(i)$, and $\in_i$ is the error term.[2] Our econometric target is to estimate *the in-group social interaction effect* $\lambda_1\in\mathbb{R}$ as well as *the out-group social interaction effect* $\lambda_2\in\mathbb{R}$.[3]

To facilitate our discussion, we rewrite Equation (1) in its equivalent matrix form:

$$\mathbf{Y}=\lambda_1\mathbf{W}_1\mathbf{Y}+\lambda_2\mathbf{W}_2\mathbf{Y}+\mathbf{X}\beta+\in, \quad (2)$$

where $\mathbf{Y}=(y_1,\dots,y_n)^{\mathrm{T}}$, $\mathbf{X}=(x_1,\dots,x_n)^{\mathrm{T}}$ and the two adjacency matrices are given by

$$\mathbf{W}_1=\begin{bmatrix} W_1 & \mathbf{0}_{n_1\times n_2} & \dots & \mathbf{0}_{n_1\times n_K}\\ \mathbf{0}_{n_2\times n_1} & W_2 & \dots & \mathbf{0}_{n_2\times n_K}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0}_{n_K\times n_1} & \mathbf{0}_{n_K\times n_2} & \dots & W_K\end{bmatrix}\in\mathbb{R}^{n\times n},$$

and

[1]A $n\times n$ adjacency matrix W is defined as follows. If i and j are connected, then $W_{ij}=1$, otherwise $W_{ij}=0$.
[2]It is a convention in the literature of social interaction models to assume that the individual characteristics X are nonstochastic; seeLee (2004) andLee (2007), among many others.
[3]In the example of student's academic achievement, y_i will be student i's GPA, x_i will be a vector of exogenous variables that may affect student's academic achievement, such as age and parents' education. $\sum_{j\in G(i)}W_{G(i),ij}y_j$ is the average GPA of student i's connected friends, and $\bar{y}_{-G(i)}$ is the average GPA outside student i's classroom.

$$\mathbf{W}_2 = \begin{bmatrix} \mathbf{0}_{n_1\times n_1} & \frac{1}{n-n_1}\mathbf{1}_{n_1}\mathbf{1}_{n_2}^{\mathrm{T}} & \cdots & \frac{1}{n-n_1}\mathbf{1}_{n_1}\mathbf{1}_{n_K}^{\mathrm{T}} \\ \frac{1}{n-n_2}\mathbf{1}_{n_2}\mathbf{1}_{n_1}^{\mathrm{T}} & \mathbf{0}_{n_2\times n_2} & \cdots & \frac{1}{n-n_2}\mathbf{1}_{n_2}\mathbf{1}_{n_K}^{\mathrm{T}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n-n_K}\mathbf{1}_{n_K}\mathbf{1}_{n_1}^{\mathrm{T}} & \frac{1}{n-n_K}\mathbf{1}_{n_K}\mathbf{1}_{n_2}^{\mathrm{T}} & \cdots & \mathbf{0}_{n_K\times n_K} \end{bmatrix} \in \mathbb{R}^{n\times n},$$

where $\mathbf{0}_{m\times n}$ denotes a $m \times n$ matrix of zeros, and $\mathbf{1}_m$ denotes a $m \times 1$ vector of ones. It is noteworthy that $\mathbf{W}_1$ and $\mathbf{W}_2$ are two $n \times n$ adjacency matrices that correspond to the in-group and out-group social interaction effects, respectively.

Remark 1: If $\lambda_2 = 0$, then Equation (1) becomes a simplified version of the models studied in Bramoullé, Djebbari, and Fortin (2009) andLee (2007). The main difference is that we do not include group-specific fixed effects here for the sake of simplicity.[4]

The microfoundation

In this subsection, we present a network game with limited information of outside groups as a microfoundation for Equation (1) following the literature (Bramoullé et al. 2007). Consider a network game in which each individual maximizes his utility by setting the optimal level of y_i. We assume that any individual i has full information of other individuals in his group but only knows the average value of the economic outcome outside his group, i.e. $\mathcal{F}_i = \{\pi_i, W_{G(i)}, \mathbf{Y}_{G(i)}, \mathbf{X}_{G(i)}, \bar{y}_{-G(i)}\}$, where π_i is the individual-specific heterogeneity in marginal return of y_i, $\mathbf{Y}_{G(i)}$ is a $n_{G(i)}$-dimensional vector of economic outcomes of the group $G(i)$ and $\mathbf{X}_{G(i)}$ is defined in the similar fashion. Each individual i is supposed to have the following utility function:

$$u_i(y_i;\mathcal{F}_i) = \underbrace{(\pi_i + \lambda_1 \sum_{j\in G(i)} W_{G(i),ij} y_j + \lambda_2\bar{y}_{-G(i)})y_i}_{\text{benefit}} - \underbrace{\frac{1}{2}y_i^2}_{\text{cost}}, \quad (3)$$

where the term $(\pi_i + \lambda_1 \sum_{j\in G(i)} W_{ij}^{G(i)} y_j + \lambda_2\bar{y}_{-G(i)})$ measures the marginal return of y_i. It is noteworthy that individual's marginal return now depends not only on his in-group friends but also the average value of the economic variable outside his group. From the first order condition, the individual i's best response function is given by

$$y_i = \pi_i + \lambda_1 \sum_{j\in G(i)} W_{G(i),ij} y_j + \lambda_2\bar{y}_{-G(i)}. \quad (4)$$

If we let $\pi_i = x_i^{\mathrm{T}}\beta + \in_i$, the best response function (4) becomes the econometric model (1). We next characterize the unique interior Nash equilibrium of the network game defined above.

Assumption 1. *The adjacency matrix W_k is row-normalized with $W_{k,ij} \geq 0$, $W_{k,ii} = 0$ for $k = 1,\ldots,K$ and $1 \leq i \leq j \leq n_k$.*

Assumption 2. $|\lambda_1| + |\lambda_2| < 1$.

Assumption 1 is standard in the literature of social interaction models (e.g.Lee 2004; Bramoullé, Djebbari, and Fortin 2009; Liu and Lee 2010). Assumption 1 requires that the group-specific adjacency matrices to be row-normalized and individuals do not link to themselves. Assumption 2 restricts the sum of the absolute values of the in-group and out-group social interaction effects, which ensures the Nash equilibrium of the network game is unique.

Proposition 1. *If Assumptions 1 and 2 hold, the matrix $(\mathbf{I} - \lambda_1\mathbf{W}_1 - \lambda_2\mathbf{W}_2)$ is invertible and the network game with payoff function (3) has a unique interior Nash equilibrium in pure strategies*:

$$\mathbf{Y} = (\mathbf{I} - \lambda_1\mathbf{W}_1 - \lambda_2\mathbf{W}_2)^{-1}\Pi,$$

where $\Pi = (\pi_1,\ldots,\pi_n)^{\mathrm{T}}$.

Proof: See the online appendix.

[4]The identification results can be derived similarly for models with fixed effects but estimation procedure would be much more complicated; see Lee (2007) for more details. We leave Equation (1) with group-specific fixed effects as a future research direction.

III. Identification and estimation

Identification

In this subsection, we show that the parameters in Equation (1) are identified under a set of mild assumptions. Let $\theta = (\lambda_1, \lambda_2, \beta^T)^T$ denote the vector of true parameters.

Assumption 3. *$\beta_i \neq 0$ for all $i = 1, \ldots, p$.*

Assumption 4. *For $i = 1, \ldots, n$, $\in_i$ is i.i.d distributed with $\mathbb{E}[\in_i] = 0$ and $\mathrm{Var}(\in_i) = \sigma^2_\in < \infty$.*

Assumption 3 ensures that all individual characteristics can be used as valid instrumental variables. Assumption 4 requires that the error terms are *i.i.d*. Both assumptions have been made in most previous studies (Kline and Tamer 2020). The next proposition establishes the identifiability of the parameters.

Proposition 2. *If Assumptions 1, 2, 3 and 4 hold, the parameters of interest $\theta = (\lambda_1, \lambda_2, \beta^T)^T$ are identified.*

Proof: See the online appendix.

Estimation

We next discuss the estimation of Equation (1). Given the fact that the OLS estimators are inconsistent because of the famous reflection problem (Manski 1993), we propose to estimate the parameters using the 2SLS method developed in Kelejian and Prucha (1998). Let $\mathbf{Z} = (\mathbf{W}_1\mathbf{Y}, \mathbf{W}_2\mathbf{Y}, \mathbf{X})$ denote the design matrix of Equation (2) and $\mathbf{H}$ denote the matrix of instrumental variables, for example, $\mathbf{H} = (\mathbf{W}_1\mathbf{X}, \mathbf{W}_2\mathbf{X}, \mathbf{X})$. The 2SLS estimators are then given by:

$$\theta_{2SLS} = (\mathbf{Z}^T\mathbf{P}_H\mathbf{Z})^{-1}\mathbf{Z}^T\mathbf{P}_H\mathbf{Y}, \tag{5}$$

where $\mathbf{P}_H = \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T$. Next, we establish the asymptotic properties of the proposed 2SLS estimators.

Assumption 5. *There exists a generic positive constant c and s_k such that $\lim_{n\to\infty} \frac{n_k}{n} = s_k > c$ for all $k = 1, \ldots, K$.*

Assumption 6. *The column sums of the group-specific adjacency matrices W_k, $k = 1, \ldots, K$ are bounded uniformly.*

Assumption 7. *The nonstochastic matrix $\mathbf{X}$ have full column rank and its elements are bounded in absolute values uniformly.*

Assumption 8. *The matrix of instrumental variables $\mathbf{H}$ has full column rank $k \geq p + 2$ for all n large enough. In addition, $\mathbf{H}$ consists of a subset of the linearly independent columns of $(\mathbf{X}, \mathbf{W}_1\mathbf{X}, \mathbf{W}_2\mathbf{X}, \mathbf{W}_1\mathbf{W}_1\mathbf{X}, \mathbf{W}_2\mathbf{W}_2\mathbf{X}\ldots)$, where the subset contains at least the linearly independent columns of $(\mathbf{X}, \mathbf{W}_1\mathbf{X}, \mathbf{W}_2\mathbf{X})$.*

Assumption 9. $Q_{HH} = \lim_{n\to\infty} n^{-1}\mathbf{H}'\mathbf{H}$ *exists and is finite and nonsingular. Furthermore,* $Q_{HZ} = plim_{n\to\infty} n^{-1}\mathbf{H}'\mathbf{Z}$ *exists and is finite and has full column rank.*

Assumption 5 requires that each group contains a substantial number of individuals, which is reasonable for most empirical applications. Furthermore, this condition together with Assumption 6 ensure that the matrices $\mathbf{W}_1$ and $\mathbf{W}_2$ have uniformly bounded row and column sums. Assumptions 5–9 are standard in the literature of social interaction models, e.g. Kelejian and Prucha (1998) and Liu and Saraiva (2015). It is noteworthy that the matrix of instrumental variables $\mathbf{H}$ is exogenous in nature as $\mathbf{X}$ is nonstochastic by assumption. The asymptotic distribution of the 2SLS estimators are given in Proposition 3.

Proposition 3. If the data are generated by Equation (1) and Assumptions 1–9 hold, then

$$\sqrt{n}(\theta_{2SLS} - \theta) \xrightarrow{d} N(0,\ [Q_{HZ}^T Q_{HH}^{-1} Q_{HZ}]^{-1}).$$

Notice that Q_{HZ} and Q_{HH} can be calculated directly using observed data, so it is straightforward to conduct statistical inference on λ_1 and λ_2 with the help of general t tests. It is noteworthy that a potential threat to the 2SLS estimation is the weak instruments problem (Staiger and Stock (1997)). To the best of our acknowledge, there is only limited research on the weak instruments problem in the context of social interaction models.[5] In fact, the weak instruments problem may even be more complicated in the current setting as both in-group and out-group effects are included. Therefore, we leave it as a promising direction for future research.

IV. Monte Carlo simulations

To investigate the finite sample performance of the proposed estimators, we conduct Monte Carlo simulations based on the following specification:

$$y_i = \lambda_1 \sum_{j \in G(i)} W_{G(i),ij} y_j + \lambda_2 \bar{y}_{-G(i)} + x_{i1}\beta_1 + x_{i2}\beta_2 + \in_i. \tag{6}$$

We consider two sets of parameters, which represent cases of weak out-group effect and strong out-group effect, respectively: (1) $\lambda_1 = 0.60$, $\lambda_2 = 0.20$ and $\beta_1 = \beta_2 = 1$; (2) $\lambda_1 = 0.20$, $\lambda_2 = 0.60$ and $\beta_1 = \beta_2 = 1$. The individual characteristics x_{i1} and x_{i2} are drawn from independent $N(0,2)$ distributions and the error term $\in_i$ is drawn from standard normal distributions. When implementing the 2SLS method, we let $\mathbf{H} = (\mathbf{X}, \mathbf{W}_1\mathbf{X}, \mathbf{W}_2\mathbf{X})$. We fix the group size to be 50 and consider three different settings: $n = 100, 200, 400$, which consist of $2, 4, 8$ groups, respectively. The group-specific adjacency matrices W_k are constructed following the specification in Liu and Lee (2010): for the ith row of W_k ($i = 1, \ldots, 50$), we draw a integer m_{ki} randomly from the set of integers $[0, 1, 2, 3, 4]$. If $i + m_{ki} < 50$ we set the $(i+1)$th, $\ldots$, $(i + m_{ki})$th elements of the ith row of W_k to be ones and the rest elements in that row to be zeros. Otherwise, the entries of ones will be wrapped around such that the first $(m_{ki} - 50)$ entries of the ith row will be ones. In the case of $m_{ki} = 0$, the ith row of W_k will have all zeros. We then normalize the matrix W_k by its row sums. The number of repetitions in each experiment is 1000. The simulation results are reported in Table 1.

Table 1. Finite sample performance of the 2SLS estimators (1000 draws).

	$n = 100$		$n = 200$		$n = 400$	
Parameters	Mean	S.D.	Mean	S.D.	Mean	S.D.
Case 1						
$\lambda_1 = 0.6$	0.5959	0.0362	0.6002	0.0331	0.5991	0.0098
$\lambda_2 = 0.2$	0.1876	0.0880	0.1956	0.1640	0.1892	0.1098
$\beta_1 = 1$	0.9927	0.0625	0.9993	0.0535	0.9997	0.0289
$\beta_2 = 1$	0.9958	0.0634	0.9991	0.0519	0.9994	0.0287
Case 2						
$\lambda_1 = 0.2$	0.2003	0.0236	0.2002	0.0162	0.2011	0.0117
$\lambda_2 = 0.6$	0.6066	0.1309	0.6018	0.0898	0.6018	0.0865
$\beta_1 = 1$	1.0002	0.0544	1.0007	0.0364	0.9988	0.0242
$\beta_2 = 1$	0.9991	0.0538	0.9993	0.0368	1.0003	0.0250

Table 2. Simulation results of the mis-specified model (1000 draws).

	$n = 100$		$n = 200$		$n = 400$	
Parameters	Mean	S.D.	Mean	S.D.	Mean	S.D.
Case 1: $\lambda_2 = 0.2$						
$\lambda_1 = 0.6$	0.5793	0.0688	0.5799	0.0401	0.5904	0.0207
$\beta_1 = 1$	0.9646	0.0941	0.9712	0.0596	0.9783	0.0375
$\beta_2 = 1$	0.9671	0.0920	0.9671	0.0609	0.9781	0.0392
Case 2: $\lambda_2 = 0.6$						
$\lambda_1 = 0.2$	0.3645	0.0979	0.3206	0.0908	0.2845	0.0735
$\beta_1 = 1$	1.1067	0.1801	1.0765	0.1208	1.0574	0.0888
$\beta_2 = 1$	1.1082	0.1731	1.0714	0.1194	1.0565	0.0901

The simulation results in Table 1 show that the 2SLS estimation method works well for our model as both the bias and the standard error of the estimates are relatively small compared with their true values. We next investigate the estimation bias of the in-group social interaction effect if the out-group effect is ignored. In this case, we adopt the standard 2SLS estimation method in Kelejian and Prucha (1998) for estimation and take $\mathbf{H} = (\mathbf{X}, \mathbf{W}_1\mathbf{X}, \mathbf{W}_1^2\mathbf{X})$ as instrumental variables. The estimation results are shown in Table 2.

The results in Table 2 indicate that ignoring the our-group social interaction effect will lead to substantiate estimation bias of the in-group social interaction effect. This problem is especially severe when the out-group effect is large (Case 2). In this sense, the model proposed in this paper can become an appealing choice for empiricists to deal with potential out-group social interaction effect in the data.

[5]The only reference we find is Tchuente (2019) who considers the identification and estimation of social interaction effect in the classic social interaction model, i.e. there only exists the in-group effect.

V. Conclusion

In this paper, we study a new class of social interaction models with both in-group and out-group effects. We provide a network game with limited information of outside groups, which rationalizes the econometric model. We show that the parameters of interest are identified under a set of mild conditions. We propose to estimate the model using the 2SLS method developed in Kelejian and Prucha (1998) and establish the asymptotic properties of the estimators. We investigate the finite sample properties of the 2SLS estimators through Monte Carlo simulations which show the estimation method performs very well.

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